$$lq(x,t)+\frac{1}{\pi\theta}\int_{-a}^{a} q(\xi,t)(-\ln|\xi-x|+d)\,d\xi+n_1 m(\bar{T})V(k_1+k_2\bar{T})\int_0^t q(x,\tau)\,d\tau=$$
$$=\delta(t)+\alpha(t)x-f(x)\quad(|x|\leqslant a,\ 0\leqslant t\leqslant\Theta<\infty).$$

The solution of the latter integral equation for conditions (1.12) can be obtained by using the method described in [8, 9]. Thus, for a sufficiently long time of wear, we obtain

$$q(x,t)=\bar{q}(1+3ex/a^2),$$
$$\delta^{\cdot}(t)=n_1 m(\bar{T})V(k_1+k_2\bar{T})\bar{q},\ \alpha^{\cdot}(t)=3n_1 m(\bar{T})V(k_1+k_2\bar{T})e\bar{q}/a^2.$$

In conclusion, it should be noted that the coefficients n_1 and n_2 must be determined experimentally for each specific combination of contiguous solids.

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NUMERICAL ANALYSIS OF FRACTURE IN PLATES UNDER THE ACTION OF IMPACT LOADS

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Calculation of fracture in solids with limited dimensions under the action of impact loads can be considered by formulating a macrofailure criterion. Fulfillment of such a criterion in a particle of the material signifies its breakdown. In the presence of a complex wave interference pattern in the numerical solution, such a criterion is satisfied in entire regions. This requires formulation of a model of the fractured solid in numerical calculations [1, 2].

There is another approach to calculating the disintegration of solids under detonation or impact loads, which is based on the porous solid model [3-6]. We shall write below the basic equations of a compressible elastoplastic medium with pores and investigate numerically the disintegration process in plates under the action of dynamic loads.

1. We shall assume that spherical defects with the radius α_0 exist in the solid. We introduce a spherical coordinate system with the origin in the spherical cavity, whose present radius is denoted by a. Assume that the stress $\sigma_r = -p$ acts at the distance b from the cavity. The porosity is characterized by

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$$\alpha = V/V_m = b^3/(b^3 - a^3), \tag{1.1}$$

where V is the volume of the material containing pores and V_m is the matrix volume. Let us determine the kinetic equation of pore growth, i.e., the variation of α in time under the action of the applied stress, assuming that the pores remain spherical during the deformation process.

Under the assumption of incompressibility of the matrix material, the basic system of equations in the spherical coordinate system is given by

$$\rho_m du/dt = \partial\sigma_r/\partial r + 2(\sigma_r - \sigma_\theta)/r,$$
$$\partial u/\partial r + 2u/r = 0,\ \sigma_r - \sigma_\theta = Y + \eta_0|\dot\gamma|^{n-1}\dot\gamma, \tag{1.2}$$

where u is the radial velocity component, $\overset{*}{\gamma} = \partial m/\partial r - u/r$, σ_r and σ_θ are the components of the stress tensor, Y is the yield point, η_0 and n are the material constants, and ρ_m is the density of the matrix material.

The boundary conditions assigned at the outer and inner boundaries are

$$\sigma_r(b,\ t) = -\alpha p + (\alpha - 1)p_\Gamma,\ \sigma_r(a,\ t) = -p_\Gamma, \tag{1.3}$$

where p_g is the gas pressure inside the cavity.

Integrating the second equation in (1.2) with respect to r, we obtain

$$u = C(t)/r^2. \tag{1.4}$$

After repeating the integration and introducing a new function $f(t) = -3\int\limits_0^t C(t)\,dt$, we write

$$r^3 - a_0^3 = -f(t), \tag{1.5}$$

whence, with an allowance for (1.1), we find

$$f(t) = b^3(\alpha_0 - \alpha)/\alpha = a_0(\alpha_0 - \alpha)/(\alpha_0 - 1) = a^3(\alpha_0 - \alpha)/(\alpha - 1). \tag{1.6}$$

We introduce a new function

$$\varphi = -\frac{C'(t)}{r} + \frac{1}{2}\frac{C^2(t)}{r^4},$$

and then

$$du/dt = \partial\varphi/\partial r. \tag{1.7}$$

By substituting $\partial\varphi/\partial r$ for du/dt in the first equation of system (1.2) and performing integration with respect to r, we obtain

$$\rho_m\,[\varphi(b,\ t) - \varphi(a,\ t)] = -\alpha(p - p_\Gamma) + 2\int\limits_a^b \frac{\sigma_r - \sigma_\theta}{r}\,dr. \tag{1.8}$$

Using the relationship $C(t) = -\frac{f'(t)}{3} = \frac{a_0^3}{3}\frac{\dot\alpha}{\alpha_0 - 1}$ and (1.6), we can transform, as in [6], the left-hand side of the equation to obtain

$$\rho_m\,[\varphi(b,\ t) - \varphi(a,\ t)] = \frac{a_0^3\rho_m}{3(\alpha_0 - 1)^{2/3}}\{\ddot\alpha\,[(\alpha - 1)^{-1/3} - \alpha^{-1/3}] -$$
$$-\frac{\dot\alpha^2}{6}[(\alpha - 1)^{-4/3} - \alpha^{-4/3}]\} = Q_1(\ddot\alpha,\ \dot\alpha,\ \alpha,\ \alpha_0)\,\rho_m.$$

We substitute the third equation of system (1.2) in the second term on the right-hand side and perform the integration, considering that all of the material around the pore is in the plastic region. We obtain

$$2\int\limits_a^b \frac{\sigma_r - \sigma_\theta}{r}\,dr = \pm\frac{2}{3}Y\ln\frac{\alpha}{\alpha - 1} + \frac{2\eta_0}{3n}|\dot\alpha|^{n-1}\dot\alpha\,\frac{\alpha^n - (\alpha - 1)^n}{(\alpha - 1)^n\alpha^n}$$

or, finally,

$$\rho_m Q_1(\ddot{\alpha}, \dot{\alpha}, \alpha, \alpha_0) = \pm\frac{2}{3} Y \ln\frac{\alpha}{\alpha-1} + \frac{2}{3}\frac{\eta_0}{n}|\dot{\alpha}|^{n-1}\dot{\alpha}\frac{\alpha^n-(\alpha-1)^n}{(\alpha-1)^n\alpha^n} - \alpha(p-p_g). \tag{1.9}$$

The term on the left-hand side of (1.9) represents the inertial resistance to changes in α; according to the estimates given in [6], it is smaller than the terms on the right-hand side by a large factor.

We introduce the quantity $\Delta p = p \pm \frac{2}{3}\frac{Y}{\alpha}\ln\frac{\alpha}{\alpha-1}$, assuming that $p_g = 0$, and we then obtain the following from (1.8) and (1.9):

$$\dot{\alpha} = \begin{cases} \left[\frac{3n}{2\eta_0}(-\Delta p)\right]^{\frac{1}{n}}(\alpha-1)\left[\frac{\alpha}{1-\left(\frac{\alpha-1}{\alpha}\right)^n}\right]^{\frac{1}{n}} & \text{for} \quad \dot{\alpha}>0, \ \Delta p<0, \\ \left[\frac{3n}{2\eta_0}\Delta p\right]^{\frac{1}{n}}(\alpha-1)\left[\frac{\alpha}{1-\left(\frac{\alpha-1}{\alpha}\right)^n}\right]^{\frac{1}{n}} & \text{for} \quad \dot{\alpha}<0, \ \Delta p>0. \end{cases} \tag{1.10}$$

The quantities η_0, n, and Y are the constants of the material. Relationship (1.10) is the kinetic equation for determining α. When α reaches a certain critical value α_*, the material fails.

2. We readily satisfy ourselves [7, 8] that, in the presence of gas in the pores p_g, the components of the stress tensors in the matrix σ_{ij}^m and the porous medium σ_{ij} are related by the expression

$$\sigma_{ij}^m = \alpha\sigma_{ij} + (\alpha-1)p_g\delta_{ij}. \tag{2.1}$$

Representing the stress tensor in the matrix and the porous solid in the form of the spherical and the deviator parts, we write

$$p^m = \alpha p - (\alpha-1)p_r, \quad s_{ij}^m = \alpha s_{ij}. \tag{2.2}$$

On the basis of (2.2), we shall express the Mises yield criterion for the matrix material in terms of the deviator components of the stress tensor for a porous medium in the following form:

$$s_{ij}s_{ij} = [Y(p, T)/\alpha]^2. \tag{2.3}$$

Assuming that the principle of minimum work of the actual stresses at plastic strain increments holds for a porous solid [9], we write an expression relating the components of the deviator of the strain rate tensor and the deviator of the stress tensor which is similar to that given in [10],

$$2\mu\dot{e}_{ij} = D/Dts_{ij} + \lambda s_{ij}, \tag{2.4}$$

where D/Dt denotes a derivative in the sense of Jaumann, $\dot{e}_{ij}$ are the tensor components of the strain rate deviator in a porous medium, and μ is the sheer modulus.

The value of λ is determined by means of condition (2.3), and, in the absence of plastic strain, $\lambda \equiv 0$. The expression for λ can readily be obtained by multiplying both sides of (2.4) by s_{ij}. In accordance with [11], the following relationships can be established between the components of the deviators of the strain rate tensors and the first invariants of the strain rate tensors in the matrix and the porous solid:

$$\dot{e}_{ij}^m = \dot{e}_{ij}\left(1-\frac{\dot{\alpha}}{\alpha}\Theta^{-1}\right), \quad \dot{\Theta}^m = \dot{\Theta} - \frac{\dot{\alpha}}{\alpha}. \tag{2.5}$$

In order to determine the spherical component of the stress tensor in the porous solid, we use the equation of state for the matrix material in the Mie–Grueneisen form:

$$p^m = Q(V_m) + \gamma_m E_m/V_m. \tag{2.6}$$

The pressure in a porous solid changes due to variations in the specific volume and specific energy of the material and also variations in the pore volume or, which is the same, changes in α. Let us determine the pressure increment in the porous solid, assuming that $p_g = 0$ and $\gamma_m/V_m = \gamma_0/V$ = constant neglecting the increment in the surface energy of pores:

$$dp = -\frac{d\alpha}{\alpha^2}\left[Q(V_m) + \frac{\gamma_0}{V_0}E_m\right] + \frac{1}{\alpha}\left[\frac{\partial Q}{\partial V_m}dV_m + \frac{\gamma_0}{V_0}dE_m\right]. \tag{2.7}$$

Finally, we rewrite (2.7) in the following form:

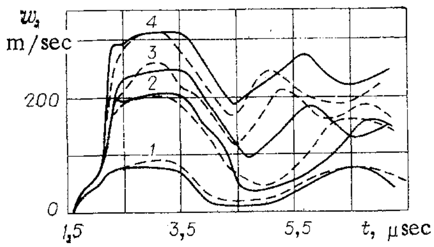

Fig. 1

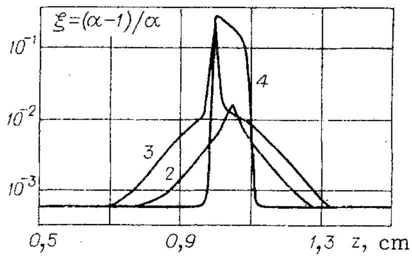

Fig. 2

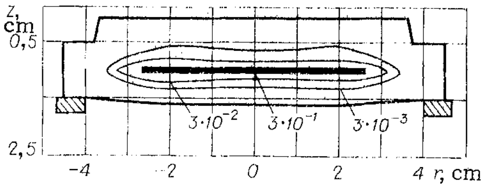

Fig. 3

$$dp = -\frac{d\alpha}{\alpha}p + \frac{1}{\alpha}\left[\frac{\partial Q}{\partial V_m}\left(\frac{dV}{\alpha} - V\frac{d\alpha}{\alpha^2}\right) + \frac{\gamma_0}{V_0}dE\right], \tag{2.8}$$

where p, V, and E are the pressure, volume, and internal energy of the porous solid, respectively. This expression constitutes the differential relationship for determining the value of p in the porous medium.

The equations of motion of the porous continuous medium are written as follows:

$$\frac{\partial}{\partial t}\int_V U dV + \int_S \mathbf{H}\cdot\mathbf{n}dS = 0; \tag{2.9}$$

$$U = \begin{Vmatrix} \rho u^1 \\ \rho u^2 \\ \rho u^3 \\ \rho \\ \rho e \end{Vmatrix}, \quad \mathbf{H} = \begin{Vmatrix} \hat{\sigma}\cdot\mathbf{e}_1 \\ \hat{\sigma}\cdot\mathbf{e}_2 \\ \hat{\sigma}\cdot\mathbf{e}_3 \\ 0 \\ \hat{\sigma}\cdot\mathbf{u} \end{Vmatrix}, \tag{2.10}$$

where ρ is the density, $\mathbf{u} = u^i\mathbf{e}_i$ is the velocity vector, $\mathbf{e}_i$ are unit vectors of the chosen coordinate system, $\sigma = \sigma^{ij}\mathbf{e}_i\mathbf{e}_j$ is the stress tensor, $e = E + \mathbf{u}\cdot\mathbf{u}/2$ is the total energy, and $\mathbf{n}$ is the external vector of the normal to the surface S bounding the volume V. All the quantities in (2.10) pertain to a porous medium.

Equations (2.1)-(2.5) and (2.8)-(2.10) in combination with the kinetic equation (1.10) for determining α represent a closed system of equations for the porous medium model.

3. We shall examine, within the framework of the proposed model, the numerical solutions of two-dimensional problems of collision between two disks and detonation of a cylindrical explosive charge on the surface of a plate. We shall use the experimental data from [12] in calculations pertaining to the collisional problem. We use the following physicomechanical characteristics of EI 712 steel: $\rho_0 = 7.83$ g/cm³, Y = 0.64 GPa, $\mu = 79$ GPa, n = 0.55, $\alpha_0 = 1.0006$. We use the equation of state of the matrix material in the form given in [13].

The calculation data, obtained by means of the method described in [14], are given in Figs. 1-3. Figure 1 shows the velocity of the back surface of the barrier as a function of the process time for initial impact velocities of 86, 215, 258, and 320 m/sec; these velocities pertain to curves 1-4. The dashed curves represent the experimental relationship [12]. Curve 1 indicates that there is no fracture in the barrier, while curves 2-4 reflect the development of microfractures in the material. The buildup of microfractures causes an increase in the porosity α. The breaking stress diminishes with an increase in α. The material is considered to have been destroyed when the relative volume of pores $\xi = (\alpha - 1)/\alpha$ in the cleavage plane reaches the maximum value $\xi_* = 0.3$.

Figure 2 illustrates the distribution of the relative pore volume in the barrier along the symmetry axis at the end of readings. Curves 2-4 correspond to curves 2-4 in Fig. 1.

It should be noted that during the collision between disks at an initial velocity of 215 m/sec a region of material where pores were activated and were growing ($\alpha > \alpha_0$) appeared at the center of the barrier. However, the relative pore volume in this region was insufficient for fracture to take place.

The porosity increases as the impact velocity in this zone increases. For an impact velocity of 258 m/sec, the region in the material where the pores are activated occupies more than one-half of the barrier. Maximum porosity occurs only at the target center. A further increase in the impact velocity results in a reduction of the material zone where $\alpha > \alpha_0$ and an increase in the zone where ξ reaches its maximum value.

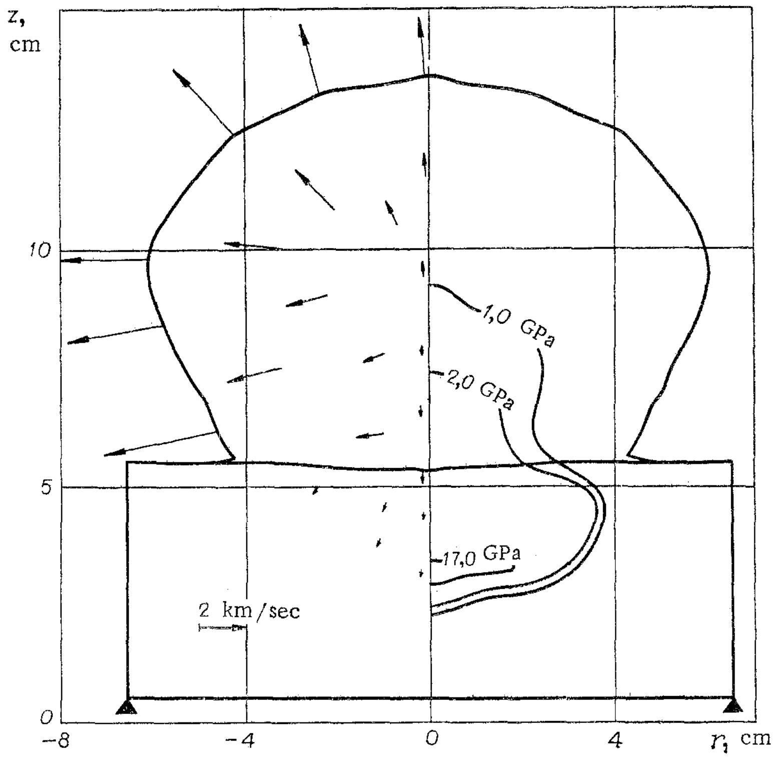

Fig. 4

Figure 3 shows the deformation and disintegration pattern of the disks at 6 μsec for an initial impact velocity of 258 m/sec. The solid curves represent the isolines of the relative pore volume. The maximum porosity occurs in a row of cells located at the barrier center in a direction perpendicular to the direction of impact, where the principal crack develops. The location of the crack coincides with its location determined in experiments [12].

We shall use the numerical method described in [15] for solving the problem of explosive charge blasting at the surface of a plate. For $\alpha_0 = \alpha = 1$, $Y = \mu = 0$, the system of equations (2.9) and (2.10) is transformed into a system describing ideal gas behavior.

The equation of state of the detonation products is given by

$$p = \gamma_1 E\rho + B_1\rho^4 + C_1 \bar{e}^{\beta/\rho},$$

where γ_1, B_1, C_1, and β are the constants characterizing the explosive [16].

The Chapman–Jouguet parameters were assigned at the front of the detonation wave in calculating the detonation of the explosive. The sliding algorithm proposed in [15] was used at the contact boundary between the metal and the detonation products. We shall use the experimental data from [17] for specific calculations.

Figure 4 shows the pattern of scattering of the detonation products at 11 μsec and provides the isobars in the plate and the detonation products. The arrows indicate the direction of scattering of the detonation products. The arrow length characterizes the velocity on a chosen scale.

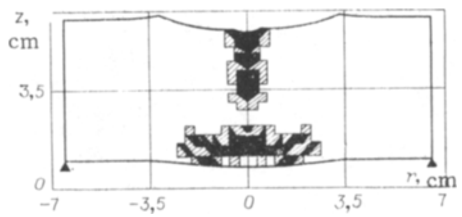

Fig. 5

The final pattern of plate deformation and disintegration that we obtained (Fig. 5) agrees with the experimental pattern [17]. The black zones denote the regions in the material where ξ has reached its maximum value ξ_*, while the hatched areas denote zones where $\alpha > \alpha_0$, but $\xi < \xi_*$.

Two disintigration zones are discernible in both calculations and experiments. The principal zone is located at the back surface of the plate (zone of rear spallation). The other zone of fractured material is located under the crater.

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